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RIGHT-HANDED NEUTRINO CURRENTS IN THE $\mathbf{SU(3)}_L \otimes \mathbf{U(1)}_N$ ELECTROWEAK THEORY

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Abstract

A version of the $\mathrm{SU(3)}_L \otimes \mathrm{U(1)}_N$ electroweak theory in which there are right-handed neutrino currents is reconsidered in detail. We argue that in order to have a result consistent with low-energy one, the right-handed neutrino component must be treated as correction instead of an equivalent spin state. The data from the Z-decay allow us to fix the limit for ϕ as $-0.00285 \leq \phi \leq 0.00018$. From the neutrino neutral current scattering, we estimate a bound for the new neutral gauge boson Z^2 mass in the range of 400 GeV. A bound for the new charged and neutral (non-Hermitian) gauge bosons $Y^{\pm}, X^o$ is also obtained from symmetry-breaking hierarchy.

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I. Introduction

The standard model (SM) [1] is in great agreement with present experimental data. Nevertheless, the belief that some questions remain unanswered, has resulted in numerous attempts to find a more fundamental underlying theory. Therefore many theories beyond the SM have been proposed [2], and their consequences are now under intensive study.

In the SM, each generation of fermions is anomaly free. This is true for many extensions of the SM as well, including the popular grand unified models [3]. In these models, therefore, the number of generations is completely unrestricted on theoretical grounds.

Recently, an interesting class of alternative models has been proposed [4] in which each generation is anomalous but different generations are not exact replicas of one another, and the anomalies cancel when a number of generations are taken into account and to be a multiple of 3. The most economical gauge group which admits such fermion representations is $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$, and it has been proposed by Pisano, Pleitez and Frampton [5] (for further work on this model see Refs. [6, 7]). The original model did not have right-handed (RH) neutrinos, but in [8] they have been included. However, by some reasons, consequences of this model (model I as called in [8]) are too different from those of the SM. For example, *the magnitude of the neutral couplings of the right-handed neutrinos coincides with that of the left-handed neutrinos in the SM* (see Eqs.(44,51) in [8]). If so, in order to get results consistent with low energy phenomenology such as $\nu_\mu - e$ and $\bar{\nu}_\mu - e$ scatterings, $g_L(e)$ must be replaced by $g_R(e)$ and vice versa [10]. It is obvious that this statement is unacceptable.

The purpose of this paper is to reconsider this model. It is to be noted that a similar model has been proposed in [11] (for details see [12, 13]) in which RH neutrinos by opposition to our model, gives contribution to neutral currents of the left-handed (LH) neutrinos. Since there are many analogies between the two models in the Higgs and gauge bosons sectors, we will briefly present them. The main difference arises in neutrino neutral current scattering, which we will consider in the Section V.2. By introducing the $Z - Z'$ mixing angle, the exact physical eigenstates of neutral gauge bosons are obtained. Based on the data from the Z-decay, the mixing angle is fixed and from the neutrino neutral current scattering data, we estimate the Z^2 boson mass in the range of 400 GeV, which is accessible for direct searches at high energy colliders such as Tevatron, NLC, etc.

This paper is organized as follows. In Sec. II we give a brief review of the model. The charged and neutral currents are studied in Sec. III. In Sec. IV the constraints on the $Z - Z'$ mixing and masses of the new gauge bosons are obtained. Here we argue that in order to have a result consistent with low energy one, the RH neutrino component must be treated as correction instead of an equivalent spin state. Finally, our conclusions are summarized in the last section.

II. The model

Our model is based on the gauge group

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_N. \tag{1}$$

1. Fermion content and Yukawa interactions

This model deals with nine leptons and nine quarks. There are three left- and right-handed neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$, three charged leptons (e, μ, τ), four quarks with charge 2/3, and five quarks with charge -1/3.

Under the gauge symmetry (1), the three lepton generations transform as

$$f_L^a = \begin{pmatrix} \nu_L^a \\ e_L^a \\ (\nu_L^c)^a \end{pmatrix} \sim (1, 3, -1/3), e_R^a \sim (1, 1, -1), \tag{2}$$

where a = 1, 2, 3 is the generation index.

Two of the three quark generations transform identically and one generation (it does not matter which one) transforms in a different representation of the gauge group (1):

$$Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ d'_{iL} \end{pmatrix} \sim (3, \bar{3}, 0), \tag{3}$$

$$u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), d'_{iR} \sim (3, 1, -1/3), \ i = 1, 2,$$

$$Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 1/3),$$

$$u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3).$$

Fermion mass generation and symmetry breaking can be achieved with just three $SU(3)_L$ triplets

$$\chi = \begin{pmatrix} \chi^o \\ \chi^- \\ \chi^{,o} \end{pmatrix} \sim (1, 3, -1/3), \rho = \begin{pmatrix} \rho^+ \\ \rho^o \\ \rho^{,+} \end{pmatrix} \sim (1, 3, 2/3), \ \eta = \begin{pmatrix} \eta^o \\ \eta^- \\ \eta^{,o} \end{pmatrix} \sim (1, 3, -1/3). \tag{4}$$

They are defined by Yukawa Lagrangians as follows:

$$\begin{aligned}
\mathcal{L}^\chi_{Yuk} &= \lambda_1 \bar{Q}_{3L} T_R \chi + \lambda_{2ij} \bar{Q}_{iL} d^{'}_{jR} \chi^* + \text{h.c.} \\
&= \lambda_1 (\bar{u}_{3L} \chi^o + \bar{d}_{3L} \chi^- + \bar{T}_L \chi^{,o}) T_R + \lambda_{2ij} (\bar{d}_{iL} \chi^{o*} - \bar{u}_{iL} \chi^+ + \bar{d}'_{iL} \chi^{,o*}) d'_{jR} + \text{h.c.} \\
\mathcal{L}^\eta_{Yuk} &= \lambda_{3a} \bar{Q}_{3L} u_{aR} \eta + \lambda_{4ia} \bar{Q}_{iL} d_{aR} \eta^* + \text{h.c.} \\
&= \lambda_{3a} (\bar{u}_{3L} \eta^o + \bar{d}_{3L} \eta^- + \bar{T}_L \eta^{,o}) u_{aR} + \lambda_{4ia} (\bar{d}_{iL} \eta^{o*} - \bar{u}_{iL} \eta^+ + \bar{d}'_{iL} \eta^{,o*}) d_{aR} + \text{h.c.} \\
\mathcal{L}^\rho_{Yuk} &= \lambda_{1a} \bar{Q}_{3L} d_{aR} \rho + \lambda_{2ia} \bar{Q}_{iL} u_{aR} \rho^* + G'_{ab} \bar{f}^a_L e^b_R \rho + G_{ab} \varepsilon^{ijk} (\bar{f}^a_L)_i (f^b_L)^c_j (\rho^*)_k + \text{h.c.} \\
&= \lambda_{1a} (\bar{u}_{3L} \rho^+ + \bar{d}_{3L} \rho^o + \bar{T}_L \rho^{,+}) d_{aR} + \lambda_{2ia} (\bar{d}'_{iL} \rho^- - \bar{u}_{iL} \rho^{o*} + \bar{d}'_{iL} \rho^{,-}) u_{aR} \\
&\quad + G'_{ab} [\bar{\nu}^a_L \rho^+ + \bar{e}^a_L \rho^o + (\bar{\nu}^c_L)^a \rho^{,+}] e^b_R + G_{ab} [\bar{\nu}^a_L (e^c_L)^b \rho^{,-} - \bar{e}^a_L (\nu^c_L)^b \rho^{,-}] + \text{h.c.},
\end{aligned} \tag{5}$$

here we have used: $(\bar{\nu}_L^c)^a(\nu_L^c)^b = -\bar{\nu}_R^b\nu_R^a = 0$, $(\nu_L^c)^c = -\nu_L$.

If we have the following vacuum expectation values (VEVs): $\langle\chi\rangle^T = (0,0,\omega/\sqrt{2})$, $\langle\rho\rangle^T = (0,u/\sqrt{2},0)$, $\langle\eta\rangle^T = (v/\sqrt{2},0,0)$, then all fermions gain necessary masses and the gauge symmetry is broken to the SM gauge symmetry:

$$\begin{array}{l} \mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_N \\ \quad\downarrow \langle\chi\rangle \\ \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \\ \quad\downarrow \langle\rho\rangle, \langle\eta\rangle \\ \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q. \end{array} \tag{6}$$

Here the electric charge is defined: $Q = \frac{1}{2}\lambda_3 - \frac{1}{2\sqrt{3}}\lambda_8 + N$, and the hypercharge is given: $Y = 2N - \sqrt{3}\lambda_8/3$ ($\lambda_8 = diag(1,1,-2)/\sqrt{3}$). In the present model the neutrinos remain massless at the tree level, and by radiative corrections they will gain masses [9]. In this model, the exotic quarks carry electric charges 2/3 and -1/3, respectively, similarly to ordinary quarks. Consequently, the exotic quarks can mix with the ordinary ones. This type of mixing gives the flavor changing neutral currents (FCNCs). These FCNCs will be induced due to breakdown of the GIM mechanism. This type of situation has been discussed previously and bounds on the mixing strengths can be obtained from the non-observation of FCNC's in the experiments beyond those predicted by the SM [14].

2. Gauge bosons

As usual, the covariant derivatives are

$$D_\mu = \partial_\mu + ig\sum_{a=1}^{8} W_\mu^a.\frac{\lambda_a}{2} + ig_N\frac{\lambda^9}{2}NB_\mu, \tag{7}$$

where λ^a(a=1,...,8) are the $SU(3)_L$ generators, and $\lambda^9 = \sqrt{2/3}\ diag(1,1,1)$ is defined such that $Tr(\lambda^a\lambda^b) = 2\delta^{ab}$ and $Tr(\lambda^9\lambda^9) = 2$. Also, N denotes the N charge for the three Higgs multiplets.

The non-Hermitian gauge bosons $\sqrt{2}\,W_\mu^+ = W_\mu^1 - iW_\mu^2$, $\sqrt{2}\,Y_\mu^- = W_\mu^6 - iW_\mu^7$, $\sqrt{2}\,X_\mu^0 = W_\mu^4 - iW_\mu^5$ have the following masses [12, 13]:

$$M_W^2 = \frac{1}{4}g^2(u^2+v^2), M_Y^2 = \frac{1}{4}g^2(v^2+\omega^2), M_X^2 = \frac{1}{4}g^2(u^2+\omega^2). \tag{8}$$

The physical neutral gauge bosons are defined through the mixing angle ϕ and Z, Z':

$$\begin{aligned} Z^1 &= Z\cos\phi - Z'\sin\phi, \\ Z^2 &= Z\sin\phi + Z'\cos\phi, \end{aligned} \tag{9}$$

where the photon field A_μ and Z, Z' are given by [13]:

$$\begin{aligned}
A_\mu &= s_W W^3_\mu + c_W \left(-\frac{t_W}{\sqrt{3}}\, W^8_\mu + \sqrt{1-\frac{t_W^2}{3}}\, B_\mu \right), \\
Z_\mu &= c_W W^3_\mu + s_W \left(-\frac{t_W}{\sqrt{3}}\, W^8_\mu + \sqrt{1-\frac{t_W^2}{3}}\, B_\mu \right), \\
Z'_\mu &= \sqrt{1-\frac{t_W^2}{3}}\, W^8_\mu + \frac{t_W}{\sqrt{3}}\, B_\mu. \qquad (10)
\end{aligned}$$

The mixing angle ϕ is given by

$$\tan^2\phi = \frac{M_Z^2 - M_{Z^1}^2}{M_{Z^2}^2 - M_Z^2}, \qquad (11)$$

where M_{Z^1} and M_{Z^2} are the *physical* mass eigenvalues

$$\begin{aligned}
M_{Z^1}^2 &= \frac{1}{2}\left\{ M_{Z'}^2 + M_Z^2 - [(M_{Z'}^2 - M_Z^2)^2 + 4(M_{ZZ'}^2)^2]^{1/2} \right\}, \qquad (12) \\
M_{Z^2}^2 &= \frac{1}{2}\left\{ M_{Z'}^2 + M_Z^2 + [(M_{Z'}^2 - M_Z^2)^2 + 4(M_{ZZ'}^2)^2]^{1/2} \right\}, \qquad (13)
\end{aligned}$$

with

$$\begin{aligned}
M_Z^2 &= \frac{g^2}{4c_W^2}(u^2+v^2) = \frac{M_W^2}{c_W^2}, \\
M_{ZZ'}^2 &= \frac{g^2}{4c_W^2\sqrt{3-4s_W^2}}\left[u^2 - v^2(1-2s_W^2) \right], \qquad (14) \\
M_{Z'}^2 &= \frac{g^2}{4(3-4s_W^2)}\left[4\omega^2 + \frac{u^2}{c_W^2} + \frac{v^2(1-2s_W^2)^2}{c_W^2} \right]. \qquad (15)
\end{aligned}$$

From Eq.(14) we see that $\phi = 0$ if $u^2 = v^2(1-2s_W^2)$. Here W, Z^1 correspond to the Standard Model charged and neutral gauge bosons, and there are new gauge bosons $Y^\pm, X^o$, and Z^2. A precision fit from electroweak observables gives a limit on the mixing angle (see below) of $-0.00285 \le \phi \le 0.00018$ and from the symmetry-breaking hierarchy $\omega \gg u, v$ Eq. (8) and Eq. (15) lead to

$$M_{Y^+} \simeq M_{X^o} \simeq \frac{\sqrt{3-4s_W^2}}{2} M_{Z^2} \simeq 0.72 M_{Z^2}. \qquad (16)$$

III. Charged and neutral currents

The interactions among the gauge bosons and fermions are read off directly from

$$\begin{aligned}
\mathcal{L}_F &= \bar{R} i\gamma^\mu(\partial_\mu + i g_N B_\mu N) R \\
&\quad + \bar{L} i\gamma^\mu(\partial_\mu + i\frac{g_N}{\sqrt{6}} B_\mu N + i g \sum_{a=1}^{8} W^a_\mu . \frac{\lambda_a}{2}) L, \qquad (17)
\end{aligned}$$

where R represents any right-handed singlet and L any left-handed triplet or antitriplet (here, for antitriplets, λ_a is replaced by $-\lambda_a^*$).

The interactions among the charged vector fields with leptons are

$$\begin{aligned}\mathcal{L}_l^{CC} &= -\frac{g}{\sqrt{2}}(\bar{\nu}_L^a\gamma^\mu e_L^a W_\mu^+ + (\bar{\nu_L^c})^a\gamma^\mu e_L^a Y_\mu^+ \\ &\quad +\bar{\nu}_L^a\gamma^\mu(\nu_L^c)^a X_\mu^0 + \text{h.c.}),\end{aligned} \tag{18}$$

and for the quarks we have

$$\begin{aligned}\mathcal{L}_q^{CC} &= -\frac{g}{\sqrt{2}}[(\bar{u}_{3L}\gamma^\mu d_{3L} + \bar{u}_{iL}\gamma^\mu d_{iL})W_\mu^+ + (\bar{T}_L\gamma^\mu d_{3L} + \bar{u}_{iL}\gamma^\mu d'_{iL})Y_\mu^+ \\ &\quad +(\bar{u}_{3L}\gamma^\mu T_L - \bar{d'}_{iL}\gamma^\mu d_{iL})X_\mu^0 + \text{h.c.}].\end{aligned} \tag{19}$$

We can see that the interactions with the Y^+ and X^0 bosons violate the lepton number (see Eq.(18)) and the weak isospin (see Eq.(19)).

The electromagnetic current for fermions is the usual one $Q_f e\bar{f}\gamma^\mu f A_\mu$, and the neutral current interactions can be written in the form

$$\begin{aligned}\mathcal{L}^{NC} &= \frac{g}{2c_W}\Big\{\bar{f}\gamma^\mu[a_{1L}(f)(1-\gamma_5) + a_{1R}(f)(1+\gamma_5)]fZ_\mu^1 \\ &\quad + \bar{f}\gamma^\mu[a_{2L}(f)(1-\gamma_5) + a_{2R}(f)(1+\gamma_5)]fZ_\mu^2\Big\}.\end{aligned} \tag{20}$$

The couplings of fermions with Z^1 and Z^2 bosons are given as follows:

$$\begin{aligned}a_{1L,R}(f) &= \cos\phi\,[T^3(f_{L,R}) - s_W^2 Q(f)] \\ &\quad +c_W^2\left[\frac{3N(f_{L,R})}{(3-4s_W^2)^{1/2}} - \frac{(3-4s_W^2)^{1/2}}{2c_W^2}Y(f_{L,R})\right]\sin\phi, \\ a_{2L,R}(f) &= -c_W^2\left[\frac{3N(f_{L,R})}{(3-4s_W^2)^{1/2}} - \frac{(3-4s_W^2)^{1/2}}{2c_W^2}Y(f_{L,R})\right]\cos\phi \\ &\quad +\sin\phi\,[T^3(f_{L,R}) - s_W^2 Q(f)],\end{aligned} \tag{21}$$

where $T^3(f)$ and $Q(f)$ are, respectively, the third component of the weak isospin and the charge of the fermion f. Note that for the exotic quarks, the weak isospin is equal to zero. Eqs. (21) are valid for both left- and right-handed currents. Since the value of N is different for triplets and antitriplets, the Z^2 coupling to left-handed ordinary quarks is different for the third family and thus flavor changing. Using $\bar{\nu}_L^c\gamma^\mu\nu_L^c = -\bar{\nu}_R\gamma^\mu\nu_R$ we see that in this model the neutrinos have both left-handed and right-handed neutral currents:

$$\begin{aligned}a_{1L}(\nu) &= \frac{1}{2}\left(\cos\phi + \frac{1-2s_W^2}{\sqrt{3-4s_W^2}}\sin\phi\right),\ a_{1R}(\nu) = \frac{c_W^2}{\sqrt{3-4s_W^2}}\ \sin\phi, \\ a_{2L}(\nu) &= \frac{1}{2}\left(\sin\phi - \frac{1-2s_W^2}{\sqrt{3-4s_W^2}}\cos\phi\right),\ a_{2R}(\nu) = -\frac{c_W^2}{\sqrt{3-4s_W^2}}\ \cos\phi.\end{aligned} \tag{22}$$

We can also express the neutral current interactions of Eq. (20) in terms of the vector and axial-vector couplings as follows:

$$\begin{aligned}\mathcal{L}^{NC} &= \frac{g}{2c_W}\Big\{\bar{f}\gamma^\mu[g_{1V}(f)-g_{1A}(f)\gamma_5\Big]fZ^1_\mu \\ &\quad + \bar{f}\gamma^\mu[g_{2V}(f)-g_{2A}(f)\gamma_5]fZ^2_\mu\Big\}. \end{aligned}\tag{23}$$

The values of these couplings are:

$$\begin{aligned}
g_{1V}(f) &= \cos\phi\,[T^3(f_L)-2s_W^2Q(f)] \\
&\quad + \sin\phi\left[\frac{c_W^2}{(3-4s_W^2)^{1/2}}(3N(f_L)+t_W^2N(f_R))-\sqrt{3-4s_W^2}\frac{Y(f_L)}{2}\right], \\
g_{1A}(f) &= \cos\phi\,T^3(f_L) \\
&\quad + \sin\phi\left[\frac{c_W^2}{(3-4s_W^2)^{1/2}}(3N(f_L)-t_W^2N(f_R))-\sqrt{3-4s_W^2}\frac{Y(f_L)}{2}\right], \\
g_{2V}(f) &= \cos\phi\left[\sqrt{3-4s_W^2}\frac{Y(f_L)}{2}-\frac{c_W^2}{(3-4s_W^2)^{1/2}}(3N(f_L)+t_W^2N(f_R))\right] \\
&\quad + \sin\phi\,[T^3(f_L)-2s_W^2Q(f)], \\
g_{2A}(f) &= \cos\phi\left[\sqrt{3-4s_W^2}\frac{Y(f_L)}{2}-\frac{c_W^2}{(3-4s_W^2)^{1/2}}(3N(f_L)-t_W^2N(f_R))\right] \\
&\quad + \sin\phi\,T^3(f_L).
\end{aligned}$$

In the PPF model the coupling strength of Z^2 to quarks is much stronger than that of leptons due to the factor $1/\sqrt{1-4s_W^2}$. Therefore, low-energy experiments such as neutrino-nucleus scattering and atomic parity violation measurements would be useful to further constrain the model [7]. However, from (21) it is easy to see that this does not happen in this model.

In our model the interactions with the heavy charged and neutral (non-Hermitian) vector bosons Y^+, X^o violate the lepton number and the weak isospin. Because of the mixing, the mass eigenstate Z^1 now picks up flavor-changing couplings proportional to $\sin\phi$. However, since $Z-Z'$ mixing is constrained to be very small, evidence of 3-3-1 FCNC's can only be probed indirectly via the Z^2 couplings.

Whether neutrinos have a right-handed component is still an unresolved question. It is possibly the question whose answer will provide the first evidence of physics beyond the minimal SM of particle interactions.

IV. Constraints on the $Z-Z'$ mixing angle and the Z^2 mass

There are many ways to get constraints on the mixing angle ϕ and the Z^2 mass. Below we present a simple one. A constraint on the $Z-Z'$ mixing can be obtained from the Z-decay data, and so we have to calculate the width of the Z in our model.

1. Z decay modes

As in the Ref. [13] the total Z width is given by:

$$\begin{aligned}
\Gamma_{total} &= \Gamma(Z \to all) = \frac{\bar{\rho}_1 G_F}{6\sqrt{2}\pi} M_{Z^1}^3 \Big\{ \cos^2\phi \, \Delta_{total}^{SM} \\
&+3 \sin 2\phi \left[G + \frac{\sqrt{3-4\bar{s}_W^2}}{2} - \frac{D}{4\sqrt{3-4\bar{s}_W^2}} + \delta_{QCD} \left(G - \frac{E}{4\sqrt{3-4\bar{s}_W^2}} \right) \right. \\
&+ \left. \frac{\alpha}{12\pi} \left(G - \frac{F}{4\sqrt{3-4\bar{s}_W^2}} \right) \right] + O(\sin^2\phi) \Big\}, \qquad (24)
\end{aligned}$$

where

$$\begin{aligned}
D &= 9 - \frac{56}{3}\bar{s}_W^2 + \frac{272}{9}\bar{s}_W^4; \; E = 3 - \frac{20}{3}\bar{s}_W^2 + \frac{128}{9}\bar{s}_W^4, \\
F &= 33 - \frac{332}{3}\bar{s}_W^2 + \frac{1808}{9}\bar{s}_W^4; \; G = \frac{\sqrt{3-4\bar{s}_W^2}}{18}(3 - 2\bar{s}_W^2) - \frac{(3-4\bar{s}_W^2)^{3/2}}{36},
\end{aligned}$$

and

$$\Delta_{total}^{SM} = \sum_{f=\nu,e,u,d,s,c,b} \{ [\bar{g}_V^{SM}(f)]^2 + [\bar{g}_A^{SM}(f)]^2 \} (1 + \delta_{QED}^f)(1 + \delta_{QCD}).$$

Then we get the ratio

$$\begin{aligned}
R^{331} &= \frac{\Gamma(Z \to l\bar{l})}{\Gamma_{total}} = R_l^{SM} \left\{ 1 - \frac{2\tan\phi}{\sqrt{3-4\bar{s}_W^2}} \left[1 + \frac{3\sqrt{3-4\bar{s}_W^2}}{\Delta_{total}^{SM}} \right. \right. \\
&\times \left(G + \frac{\sqrt{3-4\bar{s}_W^2}}{2} - \frac{D}{4\sqrt{3-4\bar{s}_W^2}} + \delta_{QCD} \left(G - \frac{E}{4\sqrt{3-4\bar{s}_W^2}} \right) \right. \\
&+ \left. \left. \left. \frac{\alpha}{12\pi} \left(G - \frac{F}{4\sqrt{3-4\bar{s}_W^2}} \right) \right) \right] + O(\tan^2\phi) \right\}, \qquad (25)
\end{aligned}$$

where R_l^{SM} denotes the SM result: $R_l^{SM} = 0.03362$, for $\alpha^{-1}(M_Z) = 128.87$ [15, 19], $\alpha_s(M_Z) = 0.118$, and for $\bar{s}_W^2(M_Z) = 0.2333$ [17]. Taking the experimental result in [15] $\Gamma = (3.367 \pm 0.006)\%$, we obtain bounds for the mixing angle

$$-0.00285 \leq \phi \leq 0.00018. \qquad (26)$$

As in Ref. [13] with this mixing angle, R_b in this model still disagrees with the recent experimental value $R_b = 0.2192 \pm 0.0018$ measured at LEP [16]. We hope, however, with the inclusion of new heavy particle loop effects like exotic quarks, Higgs scalars or of new box diagrams this result will be improved and consistent with the experimental data (for recent works on this direction see [21]).

2. Neutrino-electron scattering

The motivation for focusing on the neutrino neutral current scatterings is the following: From the theoretical point of view these reactions are basic processes free from the complications of strong interactions and can be used to determine the parameters of the theories. We emphasize that in the PPF model, these processes are almost the same as in the SM (for this purpose only neutrino-nucleus scattering and atomic parity violation, etc, are suitable). Since in this model neutrinos have both left- and right-handed current, the effective four-fermion interactions relevant to ν-fermion neutral current processes, are presented as follows:

$$\begin{aligned} -\mathcal{L}^{\nu f}_{eff} &= \frac{2\rho_1 G_F}{\sqrt{2}}\Big\{g_{1V}(\nu)\bar{\nu}\gamma_\mu(1-r\gamma_5)\nu\bar{f}\gamma^\mu[g_{1V}(f)-g_{1A}(f)\gamma_5]f \\ &\quad + \xi g_{2V}(\nu)\bar{\nu}\gamma_\mu(1-r'\gamma_5)\nu\bar{f}\gamma^\mu[g_{2V}(f)-g_{2A}(f)\gamma_5]f\Big\}, \end{aligned} \tag{27}$$

where $\xi=\frac{M^2_{Z_1}}{M^2_{Z_2}}$, and $r=\frac{g_{1A}(\nu)}{g_{1V}(\nu)}, r'=\frac{g_{2A}(\nu)}{g_{2V}(\nu)}$ are *right-handedness* of currents.

The Feynman amplitude for the $\nu_\mu - e$ scattering is

$$\begin{aligned} T_{if} &= \frac{2\rho_1 G_F}{\sqrt{2}}\{\bar{\nu}(k')\gamma_\mu(1-r\gamma_5)\nu(k)\bar{e}(p')\gamma^\mu[g_{1V}(\nu)g_{1V}(e)-g_{1V}(\nu)g_{1A}(e)\gamma_5]e(p) \\ &\quad + \xi\bar{\nu}(k')\gamma_\mu(1-r'\gamma_5)\nu(k)\bar{e}(p')\gamma^\mu[g_{2V}(\nu)g_{2V}(e)-g_{2V}(\nu)g_{2A}(e)\gamma_5]e(p)\}. \end{aligned} \tag{28}$$

In the calculation, it is convenient to introduce the following symbols:

$$\begin{aligned} & L^{\mu\nu}(l,p,p',g_V(l),g_A(l),g'_V(l),g'_A(l)) \equiv Tr\Big\{\hat{p}[g_V(l)+g_A(l)\gamma_5]\gamma^\mu\hat{p'}\gamma^\nu[g'_V(l)+g'_A(l)\gamma_5]\Big\} \\ & = 4\{[g_V(l)g'_V(l)-g_A(l)g'_A(l)][p^\mu p'^\nu + p^\nu p'^\mu - g^{\mu\nu}(p.p')] + i[g_V(l)g'_A(l)-g_A(l)g'_V(l)] \\ & \varepsilon^{\mu\nu\alpha\beta}p_\alpha p'_\beta\Big\}, \\ & l_{\mu\nu}(\nu,k,k',r,r') \equiv Tr\Big\{\hat{k}[1+r\gamma_5]\gamma_\mu\hat{k'}\gamma_\nu[1+r'\gamma_5]\Big\} \\ & = 4\Big\{[1-r(\nu)r'(\nu)][k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(k.k')] + i(r'-r)\varepsilon^{\mu\nu\alpha\beta}k_\alpha k'_\beta\Big\} \end{aligned} \tag{29}$$

which arise in many trace calculations and have the property

$$\begin{aligned} & L^{\mu\nu}(l,p,p',g_{1V}(l),g_{1A}(l),g_{2V}(l),g_{2A}(l))l_{\mu\nu}(\nu,k,k',r,r') \\ & = 32\{[g_{1V}(l)g_{2V}(l)-g_{1A}(l)g_{2A}(l)](1-rr')[(p.k)(p'.k')+(p.k')(p'.k)] \\ & + [g_{1V}(l)g_{2A}(l)-g_{1A}(l)g_{2V}(l)](r'-r)[(p.k)(p'.k')-(p.k')(p'.k)]\}. \end{aligned} \tag{30}$$

Taking the square, summing over spin, replacing the spinor products by projection operators, and neglecting the fermion mass terms, we obtain

$$\begin{aligned} |T|^2 &= g^2_{1V}(\nu)L^{\mu\nu}(e,p,p',g_{1V}(e),g_{1A}(e),g_{1V}(e),-g_{1A}(e))l_{\mu\nu}(\nu,k,k',r,-r) \\ &\quad +2\xi g_{1V}(\nu)g_{2V}(\nu)L^{\mu\nu}(e,p,p',g_{1V}(e),g_{1A}(e),g_{2V}(e),-g_{2A}(e))l_{\mu\nu}(\nu,k,k',r,-r') \\ &\quad +\xi^2 g^2_{2V}(\nu)L^{\mu\nu}(e,p,p',g_{2V}(e),g_{2A}(e),g_{2V}(e),-g_{2A}(e))l_{\mu\nu}(\nu,k,k',r',-r') \\ &= 64G^2_F[(p.k)(p'.k')(I^e+J^e)+(p.k')(p'.k)(I^e-J^e)], \end{aligned} \tag{31}$$

where

$$\begin{aligned}
I^e &= g_{1V}^2(\nu)[g_{1V}^2(e)+g_{1A}^2(e)](1+r^2)+2\xi g_{1V}(\nu)g_{2V}(\nu)[g_{1V}(e)g_{2V}(e)+g_{1A}(e)g_{2A}(e)] \\
&\quad \times(1+rr')+\xi^2 g_{2V}^2(\nu)[g_{2V}^2(e)+g_{2A}^2(e)](1+r'^2), \\
J^e &= 4r g_{1V}^2(\nu)g_{1V}(e)g_{1A}(e)+2\xi(r+r')g_{1V}(\nu)g_{2V}(\nu)[g_{1V}(e)g_{2A}(e)+g_{1A}(e)g_{2V}(e)] \\
&\quad +4\xi^2 r' g_{2V}^2(\nu)g_{2V}(e)g_{2A}(e).
\end{aligned} \tag{32}$$

In the laboratory reference frame ($\vec{p}_e=0$), the cross section is given as in Ref. [22]:

$$\frac{d\sigma(\nu_\mu e)}{dE_e}=\frac{1}{32\pi m_e E_\nu^2}\left(\frac{1}{2.s}\sum|M|^2\right), \tag{33}$$

where E_ν, E_e are the initial neutrino and final electron energies and s is the number of the neutrino states. Perform the usual manipulations [22] we get finally

$$\begin{aligned}
\sigma(\nu_\mu e) &= \frac{\rho_1^2 m_e E_\nu G_F^2}{s\pi}\left[(I^e+J^e)+\frac{1}{3}(I^e-J^e)\right], \\
\sigma(\bar{\nu}_\mu e) &= \frac{\rho_1^2 m_e E_\nu G_F^2}{s\pi}\left[\frac{1}{3}(I^e+J^e)+(I^e-J^e)\right].
\end{aligned} \tag{34}$$

It is easy to see that for $g_{1V}(\nu)=1/2, r=1$ and $\xi=0$ we get the SM results.

Substituting coupling constants into Eq.(34), we finally get

$$\begin{aligned}
\sigma(\nu_\mu e) &= \frac{\rho_1^2 m_e E_\nu G_F^2}{s6\pi}\left(\cos 2\phi+\frac{1-2s_W^2}{\sqrt{3-4s_W^2}}\sin 2\phi\right)^2 \\
&\quad \times\left\{(1-4s_W^2+8s_W^4)[(1-\xi)^2+(r-\xi r')^2]\right. \\
&\quad \left.+(1-4s_W^2)(1-\xi)(r-\xi r')\right\},
\end{aligned} \tag{35}$$

$$\begin{aligned}
\sigma(\bar{\nu}_\mu e) &= \frac{\rho_1^2 m_e E_\nu G_F^2}{s6\pi}\left(\cos 2\phi+\frac{1-2s_W^2}{\sqrt{3-4s_W^2}}\sin 2\phi\right)^2 \\
&\quad \times\left\{(1-4s_W^2+8s_W^4)[(1-\xi)^2+(r-\xi r')^2]\right. \\
&\quad \left.-(1-4s_W^2)(1-\xi)(r-\xi r')\right\}.
\end{aligned} \tag{36}$$

From Eqs.(35,36), we see that when $\xi=0$, then $\phi=0,\ r=r'=1$, and the low-energy SM results are obtained if and only if $s=1$. Note that the formulas (35,36) are valid for theories with one extra neutral gauge boson Z^2 and neutrinos having left- and right-handed components.

For this model we have:

$$\begin{aligned}
r(\nu) &\simeq 1-\frac{4c_W^2}{\sqrt{3-4s_W^2}}\tan\phi+O(\tan^2\phi), \\
r'(\nu) &\simeq -\frac{1}{3-4s_W^2}\left(1+\frac{4c_W^2}{\sqrt{3-4s_W^2}}\tan\phi\right)+O(\tan^2\phi).
\end{aligned} \tag{37}$$

As in [13], taking an average value for $\phi = -0.00134$, $m_t = 174$ GeV [19], the running $s_W^2 = 0.21$ and the experimental results on $\frac{\sigma(\bar{\nu}_\mu e)}{E_\nu} = (1.17 \pm 0.206) \times 10^{-42} cm^2/GeV$ and $\frac{\sigma(\nu_\mu e)}{E_\nu} = (1.8 \pm 0.32) \times 10^{-42} cm^2/GeV$ given in [23] the allowed range of the new gauge boson masses are $M_{Z^2} \geq 320$ GeV and 280 GeV , respectively . Thus Eq. (16) gives a limit for the masses of the gauge bosons $Y^\pm, X^o$: $M_X \geq 230$ GeV and 200 GeV, respectively.

In our model, the free parameters are $\sin^2\theta_W, M_{Z^2}$, and ϕ which are constrained from experiment. M_{Z^1} is related by Eq. (11) where $M_Z = M_W/\cos\theta_W$ is the prediction for the Z mass in the absence of mixing $\phi = 0$. It is interesting to consider the special case $\phi = 0$. We then have $M_{Z^1} = M_Z, \rho_1 = 1$ and $s_W(M_{Z^1}) = s_W(M_Z)$. From Eq.(35) and Eq.(36) we get bounds for the new gauge bosons Z^2 mass: $M_{Z^2} \geq 460$ GeV and 350 GeV, respectively. Thus, the only way to get a rigorous bound of M_{Z^2} is through low energy processes as considered in this paper. Our bounds could be improved significantly with more precise data.

V. Discussion

In this paper, we reconsidered and presented a further development of the 331 model with neutrino right-handed currents. We have shown that this model has some advantages over the original 331 model. First, in the Higgs sector, we need only three Higgs triplets for generating fermions and gauge bosons masses as well as for breaking the gauge symmetry. Moreover in the limit $\phi = 0$, all couplings of the ordinary fermions to Z^1 boson are the same as in the SM. In this model there is no limit for the Weinberg angle $\sin^2\theta_W < \frac{3}{4}$.

The lepton number is violated in the heavy charged and neutral (non-Hermitian) vector bosons interactions. We also have flavor-changing neutral currents in the quark sector coupled to the new Z^2 boson. All the heavy bosons have masses depending on $\langle\chi\rangle$ and this VEV is, in principle, arbitrary. We argue that in order to get the low energy SM result right-handed component of neutrinos in this model has to be considered as a correction instead of an equivalent spin state (spin-average factors of $\frac{1}{2}$).

Finally, we emphasize again that experimental data from the Z-decay and $\bar{\nu}_\mu - e,\ \nu_\mu - e$ scattering processes allows us to estimate the mixing angle ϕ and the new gauge boson masses. To get stronger limits we have to consider other parameters such as the left-right cross section asymmetry, N_ν, etc.

To summarize, we have shown that because of the $Z - Z'$ mixing, there is a modification to the Z^1 coupling proportional to $\sin\phi$, and the Z-decay gives $-0.00285 \leq \phi \leq 0.00018$. The data from neutrino neutral current elastic scatterings shows that mass of the new neutral gauge boson M_{Z^2} is in the range of 400 GeV (90% C.L.), and from the symmetry-breaking hierarchy we get: $M_{Y^+} \simeq M_{X^o} \simeq 0.72 M_{Z^2} \geq 290$ GeV. It may well be that the next stage of the developments will consist of the discoveries of more sequential right-handed neutrinos.

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